# Subject: Leaving Certificate Maths

# Teacher: Mr Murphy

# Lesson 20: Financial Maths II

## 20.1 Learning Intentions

**After this week's lesson you will be able to;**

- Calculate percentage error and tolerance
- Solve problems involving mark-up/margin.
- Solve problems involving compound interest/depreciation.
- Use geometric series in a financial context.
- Use the idea of present value to calculate loan repayments.
- Derive a formula for mortgage repayments.

## 20.2 Specification

- solve problems that involve
- calculating cost price, selling price, loss, discount, mark up (profit as a % of cost price), margin (profit as a % of selling price)
- compound interest, depreciation (reducing balance method), income tax and net pay (including other deductions)
- costing: materials, labour and wastage
- metric system; change of units; everyday imperial units (conversion factors provided for imperial units)
- make estimates of measures in the physical world around them
- use present value when solving problems involving loan repayments and investments

## 20.3 Chief Examiner's Report

40%). The highest mean mark was 88% in Question 3 (functions). Candidates also performed particularly well on Question 1 (sequences), Question 2 (algebra), and Question 6 (financial applications of sequences).

## 20.4 Percentage Error and Tolerance

This is described as the amount the current value differs from the true value

$$Percentage\ Error = \frac{error}{true\ value} \times 100$$


Copy down example from the video:

## 20.5 Mark-up/Margin

Mark is a direct comparison of the profit made and the cost of the item. It's given by the formula:

$$Mark\ Up = \frac{Profit}{Cost\ Price} \times 100$$

Whereas, the margin (or profit margin) is the comparison of the profit and the price the item is sold at:

$$Margin = \frac{Profit}{Sell\ Price} \times 100$$

## 20.6 Compound Interest

This is a type of interest whereby the amount the interest is worth in euro varies with the compounding period (time) but the percentage remains constant. The reason for this is that the amount we are calculating the percentage of changes.

For example, with an interest of 10% AER (**A**nnual **E**quivalent **R**ate, can also be called **E**quivalent **A**nnual **R**ate or **C**ompound **A**nnual **R**ate), €100 is saved each year. We have the following situation:

| | Start of Year | Interest (%) | Interest(€) | End of Year |
|---|---|---|---|---|
| YEAR 1 €100 | 10% | €10 | €110 | |
| YEAR 2 €110 | 10% | €11 | €121 | |
| YEAR 3 €121 | 10% | €12.10 | €133.10 | |

...and so on.

To enable a quick calculation of the final amount **(F)** including interest **(i)** added to the present value **(P)** after a number of years **(t)** we have the following formula:

$$F = P(1 + i)^t$$

### Practice Q:

i) Find the future value of €5000 at 3% AER per annum, compounded annually over 3 years. Also find the interest gained over those same 3 years.

ii) How much would you need to invest now at a rate of 4.25% per annum to have €6000 in 3 years?

## 20.7 Depreciation

This is the reduction in the value of an asset over time.
It decreases as a percentage of the value that year.
Therefore, we can say it is very similar to the idea of compound interest, thus we make a slight alteration to the compound interest formula.

$$F = P(1-i)^t$$

**We also need to be aware of two terms in the context of depreciation:**

- Net Book Value or NBV is the value of the asset after depreciation.
- Depreciation written off is the value that has been taken off the cost price of the item.

## 20.8 Interest Periods

This refers to the length of time that elapses before interest is applied. We can have annually, monthly, quarterly etc. Interest can also be compounded over various periods of time, for example if with had 3.2% AER that was compounded monthly.

So, it's 3.2% per year, so logic would suggest 3.2% divided by 12 to get the monthly rate, however just like compound interest the amount each month changes so we need to find the monthly rate that's equivalent to 3.2% AER.

$3.2\% = 0.032$

$(1+0.032)^{1/12} - 1$

$0.2628336959\%$...Equivalent Monthly Rate

## 20.9 Investments/Savings

Savings and investments could be modelled using a Geometric Series. For example, if we look at saving €50 a year for 4 years @10% AER, we end up with a situation like:

So now that you've filled in the future values of each of the yearly €50 savings, we can add them up to find the total value of the saving at the end of the 4 years.

**Total Value:**

A quicker way would be to treat the payments like a geometric series and establish the common ratio, which would allow us to use the formula:

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

### Verify your answer below using the $S_n$ formula.

Also, with investments and savings we sometimes end up with a mixture regarding the AER and compounding period, in this instance we have two options:

1) Use the AER as it is but change time to fractions.
2) Convert AER to monthly rate.

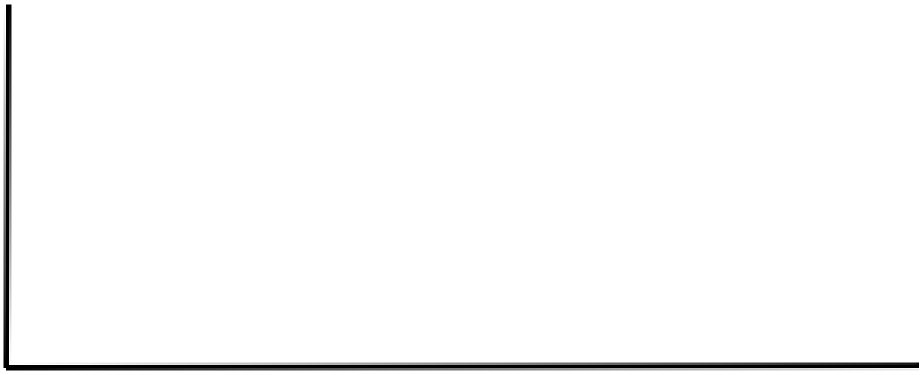
For example, I'm saving for a holiday monthly by putting away €400 into a savings account. The bank is giving an AER of 6% compounded monthly. So, if I am heading away in four years, how much will I have saved with the bank?

### Fill in the solution from the video below:

## 20.10 Mortgages

Mortgages are loans that are paid back with equal installments.
Such loans are referred to as amortised loans.

### Sketch in the graph of loan repayments below:



So to aid us with these amortised loans we have a formula known as the amortization formula (Pg 31 of Tables book) which is derived in the following manner:

We start the derivation by using our compound interest formula with a slight variation:

$F = P(1+i)^t$...$P = Sum\ of\ all\ the\ repayments\ at\ present\ (Sum\ borrowed)$

$A = P(1+i)^t$...$A = Annual\ Repayment$

$\frac{A}{(1+i)^t} = P$.... Let t be the years and start at year 1

$$P = \frac{A}{(1+i)^1} + \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3} \ldots$$

$$P = A\left(\frac{1}{(1+i)^1} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} \ldots\right)$$

$$P = A\left(\frac{a(1-r^n)}{1-r}\right)\ldots\ldots a = \frac{1}{1+i} \text{ and } r = \frac{1}{1+i} \text{ and } n = t$$

$$P = A\left[\frac{\left(\frac{1}{1+i}\right)\left(1 - \frac{1}{1+i}^{t}\right)}{\left(1 - \frac{1}{1+i}\right)}\right]$$

### Simplify by multiply out the top and get a common denominator on the bottom.

$$P = A\left[\frac{\frac{1}{1+i} - \frac{1}{(1+i)^{t+1}}}{\left(\frac{1+i-1}{1+i}\right)}\right]$$

Multiply above and below by $(1 + i)^{t+1}$

$$P = A\left[\frac{(1+i)^t - 1}{i(1+i)^t}\right]$$

Which can be rearranged to

$$A = P\left[\frac{i(1+i)^t}{(1+i)^t - 1}\right]$$

This is the ammortisation formula

## 20.11 Recap of the Learning Intentions

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## 20.12 Homework Task

a. Alex has a credit card debt of €5000. One method of clearing this debt is to make a fixed repayment at the end of each month. The amount of this repayment is 2.5% of the original debt.

i. What is the fixed monthly repayment, €A, required to pay the debt of €5000?

ii. The annual percentage rate (APR) charged on debt by the credit card company is 21.75%, fixed for the term of the debt. Find as a percentage, correct to 3 significant figures, the monthly interest rate that is equivalent to an APR of 21.75%.

iii. Assume Alex pays the fixed monthly repayment, €A, each month and does not have any further transactions on that card. Complete the table below to show how the balance of the debt of €5000 is reducing each month for the first three months, assuming an APR of 21.75%, charged and compounded monthly.

| Payment number | Fixed monthly payment, €$A$ | €$A$ | | New balance of debt (€) |
|---|---|---|---|---|
| | | Interest | Previous balance reduced by (€) | |
| 0 | | | | 5000 |
| 1 | | | 42·50 | 4957·50 |
| 2 | | | | |
| 3 | | | | |

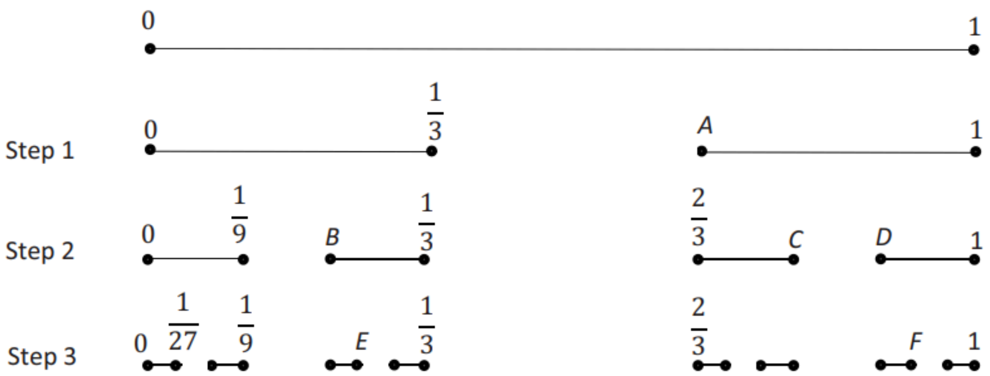
iv. Using the formula you derived on the previous page, or otherwise, find how long it would take to pay off a credit card debt of €5000, using the repayment method outlined at the beginning of part (b) above. Give your answer in months, correct to the nearest month.

v. Alex decides to borrow €5000 from the local Credit Union to pay off this credit card debt of €5000. The APR charge for the Credit Union loan is 8.5% fixed for the term of the loan. The loan is to be repaid in equal weekly repayments, at the end of each week, for 156 weeks. Find the amount of each weekly repayment.

vi. How much will Alex save by paying off the credit card debt using the loan from the Credit Union instead of paying the fixed repayment from part (b)(i) each month to the credit card company?

## 20.13 Solutions to 19.13

The process continues **indefinitely**. The set of points in the line segment [0,1] that are not removed during the process is the Contor Set.




a. (i) Complete the table below to show the lenght of the line segments (s) removed **at each step** for the first 5 **steps**. Give your answer as fractions.

| Step | Step1 | Step 2 | Step 3 | Step 4 | Step 5 |
|---|---|---|---|---|---|
| Length Removed | $\frac{1}{3}$ | $\frac{2}{9}$ | $\frac{4}{27}$ | $\frac{8}{81}$ | $\frac{16}{243}$ |

(ii) Find the **total** length of all of the line segments removed from the initial line segment of length 1 unit, after a finite number (n) of steps in the process. Give your answer in terms of n.

$$a = \frac{1}{3}, r = \frac{2}{3}$$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_n = \frac{\frac{1}{3}(1 - \left(\frac{2}{3}\right)^n)}{1 - \frac{2}{3}} = 1 - \left(\frac{2}{3}\right)^n$$

(iii) Find the total length removed, from the initial line segment, after an infinite number of steps of the process.

As this is an infinite geometric series we use the same a and r with the formula:

$$S_n = \frac{a}{1 - r} = \frac{\frac{1}{3}}{1 - \frac{2}{3}} = 1$$

$$\lim_{n \to \infty} \left(1 - \frac{2}{3}\right)^n = 1$$

b. (i) Complete the table **below** to identify the end-points labelled in the diagram. Give your answers as **fractions**.

| Label | *A* | *B* | *C* | *D* | *E* | *F* |
|---|---|---|---|---|---|---|
| End-point | $\frac{2}{3}$ | $\frac{2}{9}$ | $\frac{7}{9}$ | $\frac{8}{9}$ | $\frac{7}{27}$ | $\frac{25}{27}$ |

**(ii)** Give a reason why $\frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \frac{1}{81}$ is a point in the *Cantor Set*.

It is the end point of a segment or $\frac{7}{27} - \frac{1}{81} = \frac{20}{81}$

**(iii)** The limit of the series $\frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \cdots$ is a point in the *Cantor Set*. Find this point.

$$S_\infty = \frac{a}{1 - r}$$

$$S_\infty = \frac{\frac{1}{3}}{1 - \left(-\frac{1}{3}\right)}$$

$$S_\infty = \frac{1}{4}$$